CHAPTER 7 – THE PN JUNCTION

Valence Band

Conduction Band

E

drift (due to E-field)

diffusion

p-type

n-type

Fermi level

thermal electron-hole pair creation

eV_g

eV_j

eV_g

Schematic of pn-junction
The PN Junction

- **Chapter 4**: we considered the semiconductor in equilibrium and determined electron and hole concentrations in the conduction and valence bands, respectively.

- The net flow of the electrons and holes in a semiconductor generates **current**. The process by which these charged particles move is called **transport**.

- **Chapter 5**: we considered the two basic transport mechanisms in a semiconductor crystal: **drift**: the movement of charge due to electric fields, and **diffusion**: the flow of charge due to density gradients.
The PN Junction

- **Chapter 6**: we discussed the behavior of non-equilibrium electron and hole concentrations as functions of time and space.

- We developed the *ambi-polar transport equation* which describes the behavior of the excess electrons and holes.

- **Previous Chapters**: we have been considering the properties of the semiconductor material by calculating electron and hole concentrations in thermal equilibrium and determined the position of the Fermi level.
The PN Junction

- **Previous Chapters**: We considered the non-equilibrium condition in which excess electrons and holes are present in the semiconductor.

- **Chapter 7**: We now wish to consider the situation in which a p-type and an n-type semiconductor are brought into contact with one another to form a PN junction.
The PN Junction

- Most semiconductor devices contain at least one junction between p-type and n-type semiconductor regions.
- Semiconductor device characteristics and operation are intimately connected to these PN junctions, therefore considerable attention is devoted initially to this basic device.
- The PN junction diode provides characteristics that are used in rectifiers and switching circuits and will also be applied to other devices.
- The electrostatics of the PN junction is considered in this chapter and the current-voltage characteristics of the PN junction diode are developed in the next chapter.
Basic Structure of the PN Junction

- The entire semiconductor is a single-crystal material with one region doped with acceptor impurity atoms “p-region” and the adjacent region doped with donor atoms to form the “n-region”. The interface separating n and p regions is the **metallurgical junction**.

- We consider a step junction (the doping concentration is uniform in each region and there is an abrupt doping change at the junction.)

- Initially, at the metallurgical junction, there is a very large density gradient in both the electron and hole concentrations.
Basic Structure of the PN Junction

- **Majority carrier electrons** in the n region will begin diffusing into the p-region and **majority carrier holes** in the p-region will begin diffusing into the n region.

- As electrons diffuse from the n region, **positively charged donor atoms are left behind**. Similarly, as holes diffuse from the p region, they uncover **negatively charged acceptor atoms**.

- **The net positive and negative charges induce an electric field** in the region near the metallurgical junction from the positive to the negative charge, or from the n to the p region.
Basic Structure of the PN Junction

Fig. 7.1: Basic Structure of the PN Junction
Basic Structure of the PN Junction

- The two regions are referred to as the **space charge region or depletion region**. Essentially all electrons and holes are swept out of the space charge region by the electric field.

- Density gradients still exist in the majority carrier concentrations at each edge of the space charge region. We can think of a density gradient as producing a "**diffusion force**" that acts on the majority carriers.

- The **electric field in the space charge region** produces another force on the electrons and holes which is in the **opposite direction to the diffusion force** for each type of particle. **In thermal equilibrium, the diffusion force and the E-field force exactly balance each other.**
Fig. 7.2: Flat Band Diagram of a PN Junction
Zero applied bias

- The properties of the step junction in thermal equilibrium, the space charge region width, the electric field, and the potential through the depletion region where no currents exist and no external excitation is applied are studied.

- Assuming no voltage is applied across the PN junction, then the junction is in thermal equilibrium and the Fermi energy level is constant.

- The conduction and valance band energies must bend as we go through the space charge region, since the relative position of the conduction and valence bands with respect to the Fermi energy changes between p and n regions.

- Electrons in the conduction band of the n region see a potential barrier in trying to move into the conduction band of the p region. This barrier is the built-in potential barrier and is denoted by $V_{bi}$.

- The potential $V_{bi}$ maintains equilibrium, therefore no current is produced by this voltage.
PN Junction in Thermal Equilibrium

Fig. 7.3: PN Junction in Thermal Equilibrium
Zero applied bias

The intrinsic Fermi level is *equidistant* from the conduction band edge through the junction, thus the built-in potential barrier can be determined as the *difference* between the intrinsic Fermi levels in the p and n regions.

\[ V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| \]

In the n region, the electron concentration in the conduction band is given by:

\[ n_0 = N_c \exp \left[ -\frac{(E_c - E_F)}{kT} \right] \]

\[ n_0 = n_i \exp \left[ -\frac{(E_F - E_{Fi})}{kT} \right] \]

where \( n_i \) and \( E_{Fi} \) are the intrinsic carrier concentration and the intrinsic Fermi energy respectively.
Zero applied bias

The potential in the n region can be defined as:

\[ e | \phi_{Fn} | = E_{Fi} - E_F \]

\[ n_0 = n_i \exp\left[ -\frac{(e \phi_{Fn})}{kT} \right] \]

\[ \phi_{Fn} = -\frac{kT}{e} \ln \left( \frac{N_d}{n_i} \right) \]

Similarly, in the p region, the hole concentration is given by:

\[ e | \phi_{Fp} | = E_{Fi} - E_F \]

\[ n_0 = n_i \exp\left[ -\frac{(e \phi_{Fp})}{kT} \right] \]

\[ \phi_{Fp} = \frac{kT}{e} \ln \left( \frac{N_a}{n_i} \right) \]

Therefore, the built-in potential voltage is calculated as:

\[ V_{bi} = | \phi_{Fn} | + | \phi_{Fp} | = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \]
Zero applied bias

- At this time, we should note a **subtle but important point** concerning notation.
- Previously in the discussion of a semiconductor material, $N_d$ and $N_a$ denoted donor, and acceptor impurity concentrations in the same region, thereby forming a compensated semiconductor.
- From this point on, $N_d$ and $N_a$ will denote the **net donor and acceptor concentrations** in the individual n and p regions, respectively. If the p region, for example, is a compensated material, then $N_a$ will represent the difference between the actual acceptor and donor impurity concentrations.
- The parameter $N_d$ is defined in a similar manner for the n region.
Electric field

- Electric field is created in the depletion region by the separation of positive and negative space charge densities.
- We will assume that the space charge region abruptly ends in the n region at \( x = +x_n \) and abruptly ends in the p region at \( x = -x_p \) (\( x_p \) is a positive quantity).
Fig. 7.4:  
(a) Charge density in a p-n junction, b) Electric Field, c) Potential  
d) Energy band Diagram
Electric field

- The electric field is determined from Poisson's equation which, for a one dimension analysis, is:

\[
\frac{d^2 \phi(x)}{dx^2} = - \frac{\rho(x)}{\varepsilon_s} = - \frac{dE(x)}{dx}
\]

where \( \phi(x) \) is the electric potential, \( E(x) \) is the electric field, \( \rho(x) \) is the volume charge density, and \( \varepsilon_s \) is the permittivity of the semiconductor. The charge densities are:

\[
\rho(x) = -eN_a : -x_p < x < 0 \quad \rho(x) = -eN_d : 0 < x < x_n
\]

- The electric field in the p region is found by integrating Poisson's eqn:

\[
E = \int \frac{\rho(x)}{\varepsilon_s} dx = - \int \frac{eN_a}{\varepsilon_s} dx = - \frac{eN_a}{\varepsilon_s} x + C_1 = \frac{-eN_a}{\varepsilon_s} (x + x_p)
\]

- The electric field is zero in the neutral p region for \( x < -x_p \). As there are no surface charge densities within the PN junction structure, the electric field is a continuous function. The constant of integration is determined by setting \( E = 0 \) at \( x = -x_p \). For the n-region:

\[
E = \int \frac{\rho(x)}{\varepsilon_s} dx = - \int \frac{-eN_d}{\varepsilon_s} dx = \frac{eN_d}{\varepsilon_s} x + C_1 = \frac{-eN_d}{\varepsilon_s} (x_n - x)
\]
Electric field

- The electric field is also **continuous** at the metallurgical junction, or at \( x = 0 \) therefore:
  \[
  N_a x_p = N_d x_n
  \]

- For the uniformly doped pn junction, the E-field is a linear function of distance through the junction, and the maximum (magnitude) electric field occurs at the metallurgical junction. An electric field exists in the depletion region even when no voltage is applied between the p and n regions.

  \[
  \phi(x) = \frac{eN_a}{2\varepsilon_s} (x + x_p)^2 : (-x_p \leq x \leq 0)
  \]

  \[
  \phi(x) = \frac{eN_d}{2\varepsilon_s} (x_n - x - \frac{x^2}{2})^2 + \frac{eN_a}{2\varepsilon_s} x_p^2 : (0 \leq x \leq x_n)
  \]

- Integrating the electric field to find the built-in potential:
  \[
  V_{bi} = |\phi(x = x_n)| = \frac{e}{2\varepsilon_s} (N_d x_n^2 + N_a x_p^2)
  \]
Space Charge Width

- We can determine the **distance** that the space charge region extends into the p and n regions from the metallurgical junction. This distance is known as the **space charge width**.

$$x_p = \frac{N_d x_n}{N_a}$$

$$x_n = \left[ \frac{2\varepsilon V_{bi} N_a}{e N_d} \left[ \frac{1}{N_a + N_d} \right] \right]^{\frac{1}{2}}$$

$$x_p = \left[ \frac{2\varepsilon V_{bi} N_d}{e N_a} \left[ \frac{1}{N_a + N_d} \right] \right]^{\frac{1}{2}}$$

- The total depletion, **space charge width** $W$ is the sum of the two:

$$W = \left[ \frac{2\varepsilon V_{bi}}{e} \left[ \frac{N_a + N_d}{N_a \cdot N_d} \right] \right]^{\frac{1}{2}}$$
Exercise

1. Calculate $V_{bi}$ in a Silicon PN junction at $T = 300K$ for (a) $N_d = 10^{15}$ cm$^{-3}$ and:
   i) $N_a = 10^{15}$
   ii) $N_a = 10^{16}$
   iii) $N_a = 10^{17}$
   iv) $N_a = 10^{18}$.

(b) Repeat part (a) for $N_d = 10^{18}$ cm$^{-3}$.

$$V_{bi} = \left| \phi_{Fn} \right| + \left| \phi_{Fp} \right| = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right)$$
Solution
1. Using the equation:

\[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \]

where \( V_t = 0.0259 \text{V} \) and \( n_i = 1.5 \times 10^{10} \text{cm}^{-3} \),

<table>
<thead>
<tr>
<th>For ( N_d = 10^{15} \text{cm}^{-3} )</th>
<th>( V_{bi} ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_a = 10^{15} \text{cm}^{-3} )</td>
<td>0.575V</td>
</tr>
<tr>
<td>( N_a = 10^{16} \text{cm}^{-3} )</td>
<td>0.635</td>
</tr>
<tr>
<td>( N_a = 10^{17} \text{cm}^{-3} )</td>
<td>0.695</td>
</tr>
<tr>
<td>( N_a = 10^{18} \text{cm}^{-3} )</td>
<td>0.754</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>For ( N_d = 10^{18} \text{cm}^{-3} )</th>
<th>( V_{bi} ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_a = 10^{15} \text{cm}^{-3} )</td>
<td>0.754V</td>
</tr>
<tr>
<td>( N_a = 10^{16} \text{cm}^{-3} )</td>
<td>0.814</td>
</tr>
<tr>
<td>( N_a = 10^{17} \text{cm}^{-3} )</td>
<td>0.874</td>
</tr>
<tr>
<td>( N_a = 10^{18} \text{cm}^{-3} )</td>
<td>0.933</td>
</tr>
</tbody>
</table>
2. An abrupt Silicon PN junction at zero bias has dopant concentration of $N_a = 10^{17}$ cm$^{-3}$ and $N_d = 2 \times 10^{16}$ cm$^{-3}$, $T = 300$K. (a) Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level. (b) Sketch the equilibrium energy band diagram for the junction and determine $V_{bi}$ from the diagram and the results of part (a), (c) Calculate $V_{bi}$ and compare the results to part b). (d) Determine $x_n$ and the peak electric field for this junction.

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right)$$
Solution

- (a) n-side,
  
  \[ E_F - E_{Fi} = (0.0259) \ln \left( \frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3653 \text{ eV} \]

- p-side,
  
  \[ E_{Fi} - E_F = (0.0259) \ln \left( \frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3653 \text{ eV} \]

- b)
  
  \[ V_{bi} = 0.3653 + 0.3653 = 0.7306V \]

- c)
  
  \[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.0259) \ln \left( \frac{(2 \times 10^{16})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} \right) = 0.7305V \]
Solution

- d)

\[ x_n = \left[ \frac{2(11.7)(8.85 \times 10^{-14})(0.7305)}{1.6 \times 10^{-19}} \left( \frac{2 \times 10^{16}}{2 \times 10^{16}} \right) \left( \frac{1}{2 \times 10^{16} + 2 \times 10^{16}} \right) \right]^{\frac{1}{2}} \]

\[ x_n = x_p = 0.154 \, \mu m \]

\[ |E_{\text{max}}| = \frac{eN_a x_n}{\varepsilon} = \frac{(1.6 \times 10^{-19})(2 \times 10^{16})(0.154 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} = 4.76 \times 10^4 \frac{V}{cm} \]
Reverse Applied Bias

- If we apply a potential between the p and n regions, we will no longer be in an equilibrium condition and the Fermi energy level will no longer be constant through the system.

- If a positive voltage is applied to the n-region with respect to the p-region, as the positive potential is downward, the Fermi level on the n side is below the Fermi level on the p side. The difference between the two is equal to the applied voltage in units of energy.

- The total potential barrier, indicated by $V_{total}$ has increased. The applied potential is the reverse-bias condition. The total potential barrier is now given by:

$$V_{total} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$

$$V_{total} = V_{bi} + V_R$$

- where $V_R$ is the magnitude of the applied reverse-bias voltage and $V_{bi}$ is the same built-in potential barrier defined earlier.
Reverse and Forward Applied Bias

Fig. 7.5: Energy band diagram of a PN Junction under reverse and forward bias
Space Charge Width and Electric Field

- The electric fields in the neutral P and N regions are essentially zero, or at least very small, which means that the magnitude of the electric field in the space charge region must increase above the thermal-equilibrium value due to the applied voltage.

- The electric field originates on positive charge and terminates on negative charge; this means that the number of positive and negative charges must increase if the electric field increases.

- For given impurity doping concentrations, the number of positive and negative charges in the depletion region can be increased only if the space charge width $W$ increases.

- The space charge width $W$ increases with an increasing reverse-bias voltage $V_R$. 
Space Charge Width and Electric Field

- In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier. The total space charge width in case of reverse-bias can be written as:

\[ W = \left( \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right)^{1/2} \]

\[ E_{\text{max}} = -\left( \frac{2e(V_{bi} + V_R)}{\varepsilon_s} \left( \frac{N_a N_d}{N_a + N_d} \right) \right)^{1/2} \]

\[ E_{\text{max}} = -\frac{2(V_{bi} + V_R)}{W} \]
Junction Capacitance

- Since we have a separation of positive and negative charges in the depletion region, a capacitance is associated with the PN Junction.
- An increase in the reverse-bias voltage $dV_R$ will uncover additional positive charges in the $n$ region and additional negative charges in the $p$ region. The junction capacitance is defined as:

$$C' = \frac{dQ'}{dV_R}$$

$$dQ' = eN_a dx_n = eN_a dx_p$$
Junction Capacitance

- The differential charge $dQ'$ is in units of $\text{C/cm}^2$ so that the capacitance $C'$ is in units of Farads per square centimeter ($\text{F/cm}^2$), or capacitance per unit area.

- For the total potential barrier:

$$\frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

- The junction capacitance is:

$$C' = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}} \frac{N_a}{N_d} \left[ \frac{1}{N_a + N_d} \right]$$

- Exactly the same capacitance expression is obtained by considering the space charge region extending into the p region $x_p$. The junction capacitance is also referred to as the depletion layer capacitance.
Junction Capacitance

- Comparing the following two equations:

\[
W = \left( \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right)^{\frac{1}{2}}
\]

\[
C' = \left[ \frac{e \varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{\frac{1}{2}}
\]

- we find that we can write:

\[
C' = \frac{\varepsilon_s}{W}
\]

- The above equation is the same as the capacitance per unit area of a parallel plate capacitor.

- Note that the space charge width is a function of the reverse bias voltage so that the junction capacitance is also a function of the reverse bias voltage applied to the PN Junction.
Exercise

3. Consider a Silicon PN Junction at $T = 300K$ with doping concentrations of $N_a = 10^{16} \text{ cm}^{-3}$ and $N_d = 10^{15} \text{ cm}^{-3}$. Assume that $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ and let $V_R = 5 \text{ V}$. Calculate the width of the space charge region.

\[
W = \left( \frac{2 \varepsilon_s (V_{bi} + V_R)}{e} \right) \left( \frac{N_a + N_d}{N_a N_d} \right)^{1/2}
\]

\[
V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right)
\]
Solution
3. Using the equation:

\[
W = \left( \frac{2 \varepsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right)^{\frac{1}{2}}
\]

We can calculate that \( W = 2.83\mu m \).
Non-Uniformly Doped Junctions

- In the PN Junctions considered so far, we have assumed that each semiconductor region has been uniformly doped.
- In actual PN Junctions, this isn’t always true.
- In some electronic applications, specific non-uniform doping profiles are used to obtain special PN Junction capacitance characteristics.
- Different types of doping profiles are used in:
  - Uniformly Doped Junctions
  - Linearly Graded Junctions
  - Hyper-abrupt Junctions
Linearly Doped Junctions

- Considering a uniformly doped n-type semiconductor, if we diffuse acceptor atoms through the surface, the impurity concentrations will tend to be like those shown in the Figure.
- The depletion region extends into the p and n regions from the metallurgical junction.
- The net p-type doping concentration near the metallurgical junction may be approximated as a linear function of distance from the metallurgical junction.

\[ N_d - N_a = ax \]
Fig. 7.6: Impurity concentrations of a pn junction with a non-uniformly doped p region.
Linearly Doped Junctions

Similarly, the net n-type doping concentration is also a linear function of into the n region from the metallurgical junction. This effective doping profile is referred to as a linearly graded junction.
Linearly Doped Junctions

- The point \( x = x' \) corresponds to the metallurgical junction. The space charge density can be written as \( \rho(x) = eax \) where \( a \) is the gradient of the net impurity concentration.

- The electric field and potential in the space charge region from Poisson's equation and the electric field can be found as:

  \[
  \frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s} = \frac{eax}{\varepsilon_s}
  \]

  \[
  E = \int \frac{eax}{\varepsilon_s} dx = \frac{ea}{2\varepsilon_s} (x^2 - x_0^2)
  \]

- The electric field in the linearly graded junction is a quadratic function of distance. The maximum electric field occurs at the metallurgical junction. The electric field is zero at both \( x = +x_0 \) and at \( x = -x_0 \). The electric field in a non-uniformly doped semiconductor is not exactly zero, but the magnitude of this field is small therefore \( E = 0 \) in the bulk regions.

- The potential is again found by integrating the electric field as:

  \[
  \phi(x) = -\int E dx
  \]
Linearly Doped Junctions

- If we set $\phi = 0$ at $x = -x_0$ then the potential through the junction is:
  \[
  \phi(x) = -\frac{ea}{2\varepsilon_s} \left( \frac{x^3}{3} - x_0^2 x \right) + \frac{ea}{3\varepsilon_s} x_0^3 = \frac{2}{3} \frac{eax_0^3}{\varepsilon_s} = V_{bi}
  \]

- The magnitude of the potential at $x = +x_0$ will equal the built-in potential barrier for this function. Another expression for the built-in potential barrier is:
  \[
  V_{bi} = V_t \ln\left( \frac{ax_0}{n_i} \right)^2
  \]

- If a reverse-bias voltage is applied to the junction, the potential barrier increases. The built-in potential barrier $V_{bi}$ is then replaced by the total potential barrier $V_{bi} + V_R$. Solving for $x_0$ and using the total potential barrier, we obtain:
  \[
  x_0 = \frac{3}{2} \frac{\varepsilon_s}{ea} (V_{bi} + V_R)^{1/3}
  \]

- The junction capacitance per unit area can be determined by the same method as we used for the uniformly doped junction. The junction capacitance is then:
  \[
  C' = \left\{ \frac{ea \varepsilon_s^2}{12 (V_{bi} + V_R)} \right\}^{1/3}
  \]
Linearly Doped Junctions

Fig. 7.8: Differential change in space charge width with a differential change in reverse-bias voltage for a linearly graded PN Junction.

Note that $C'$ is proportional to $(V_{bi} + V_R)^{-1/3}$ for the linearly graded junction as compared to $C'\alpha(V_{bi} + V_R)^{-1/2}$ for the uniformly doped junction. In the linearly graded junction, the capacitance is less dependent on reverse-bias voltage than in the uniformly doped junction.
Hyper-abrupt Junctions

- The uniformly doped junction and linearly graded junction are not the only possible doping profiles. The equation above is used to approximate the n-type doping over a small region near \( x = x_0 \).

\[ N = Bx^m \]

- The case of \( m = 0 \) corresponds to the uniformly doped junction and \( m = +1 \) corresponds to the linearly graded junction. The cases of \( m = +2 \) and \( m = +3 \) would approximate a fairly low-doped epitaxial n-type layer grown on a much more heavily doped n+ substrate layer.

- When the value of \( m \) is negative, we have what is referred to as a hyper-abrupt junction. In this case, the n-type doping is larger near the metallurgical junction than in the bulk semiconductor. The equation above is used to approximate the n-type doping over a small region near \( x = x_0 \).
Fig. 7.9: Generalized doping profiles of a one-sided p+n junction.
Hyper-abrupt Junctions

- The junction capacitance can be derived using the same analysis method as before and is given as:

\[ C' = \left\{ \frac{eB \varepsilon_s^{(m+1)}}{(m+2)(V_{bi} + V_R)} \right\}^{1/(m+2)} \]

when \( m \) is negative, the capacitance becomes a very strong function of reverse-bias voltage, a desired characteristic in Varactor diodes. The term Varactor comes from the words variable reactor and means a device whose reactance can be varied in a controlled manner with bias voltage.

- If a Varactor diode and an inductance are in parallel, the resonant frequency of the LC circuit and the capacitance of the diode can be written in the form:

\[ f_r = \frac{1}{2\pi \sqrt{LC}} \quad C = C_0 (V_{bi} + V_R)^{-1/(m+2)} \]

- In a circuit application, we would, in general, like to have the resonant frequency be linear function of reverse-bias voltage \( V_R \) so we need: \( C \propto V^{-2} \)

- The parameter \( m \) required is found from:

\[ \frac{1}{m + 2} = 2 \quad m = -\frac{3}{2} \]

- A specific doping profile will yield the desired capacitance characteristic.
Picture Credits


  
  [Link](http://ecee.colorado.edu/~bart/book/book/contents.htm)

- Animation of the PN Junction formation, University of Cambridge, 2013.
  
  [Link](http://www.doitpoms.ac.uk/tlplib/semiconductors/pn.php)

  
  [Link](http://wanda.fiu.edu/teaching/courses/Modern_lab_manual/pn_junction.html)