CHAPTER 6 – CARRIER GENERATION AND RECOMBINATION

- Band-to-band recombination
- Trap-assisted recombination
- Auger recombination

- Generation due to light absorption
- Ionization due to charged high-energy particles
Chapter 6 – Carrier Generation and Recombination

- **Chapter 4**: we considered the semiconductor in equilibrium and determined electron and hole concentrations in the conduction and valence bands, respectively.

- The net flow of the electrons and holes in a semiconductor will generate **currents**. The process by which these charged particles move is called **transport**.

- **Chapter 5**: we considered the two basic transport mechanisms in a semiconductor crystal: **drift** the movement of charge due to electric fields, and **diffusion** the flow of charge due to density gradients.
Chapter 6 – Carrier Generation and Recombination

- **Chapter 6**: we will discuss the behavior of non-equilibrium electron and hole concentrations as functions of time and space.
- We will develop the ambi-polar transport equation which describes the behavior of the excess electrons and holes.
- We can define two new parameters that apply to the non-equilibrium semiconductor: the quasi-Fermi energy for electrons and the quasi-Fermi energy for holes.
Carrier Generation and Recombination

- **Generation**: is the process whereby electrons and holes are created.
- **Recombination**: is the process whereby electrons and holes are annihilated.
- Any deviation from thermal equilibrium will tend to change the electron and hole concentrations in a semiconductor.
- **Examples**:
  - A sudden increase in temperature will increase the rate at which electrons and holes are thermally generated so that their concentrations will change with time until new equilibrium values are reached.
  - An external excitation, such as light (a flux of photons), can also generate electrons and holes, creating a non-equilibrium condition.
The Semiconductor in Equilibrium

Since the net carrier concentrations are independent of time in thermal equilibrium, the rate at which electrons and holes are generated and the rate at which they recombine must be equal.

Let $G_{n0}$ and $G_{p0}$ be the thermal-generation rate of electrons and holes in $#/cm^3\cdot s$. For a direct band-to-band generation, the electrons and holes are created in pairs, therefore: $G_{n0} = G_{p0}$. 
Let $R_{n0}$ and $R_{p0}$ be the recombination rates of electrons and holes, respectively, for a semiconductor in thermal equilibrium, again given in units of $#/\text{cm}^3\text{-s}$.

In direct band-to-band recombination, electrons and holes recombine in pairs, so that $R_{n0} = R_{p0}$.

In thermal equilibrium, the concentrations of electrons and holes are independent of time; therefore, the generation and recombination rates are equal, so we have $G_{n0} = G_{p0} = R_{n0} = R_{p0}$.
Excess Carrier Generation and Recombination

- Electrons in the valence band may be excited into the conduction band when, for example, high-energy photons are incident on a semiconductor.

- When this happens, not only is an electron created in the conduction band, but a hole is created in the valence band; thus an electron-hole pair is generated.

- The additional electrons and holes created are called excess electrons and excess holes.
Excess Carrier Generation and Recombination

- The excess electrons and holes are generated by an external force at a particular rate.

- Let $g_n'$ be the generation rate of excess electrons and $g_p'$ be the generation rate of excess holes in units of $#/\text{cm}^3\text{s}$.

- For the direct band-to-band generation, the excess electrons and holes are also created in pairs, so we must have $g_n' = g_p'$.

- When excess electrons and holes are created, the concentration of electrons in the conduction band and of holes in the valence band increase above their thermal equilibrium value.

- We may write $n = n_0 + \delta n$ and $p = p_0 + \delta p$. $n_0$ and $p_0$ are the thermal-equilibrium concentrations, and $\delta n$ and $\delta p$ are the excess electron and hole concentrations.
# Excess Carrier Generation and Recombination

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<td>Thermal equilibrium electron and hole concentration (independent of time and position)</td>
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<td>$n, p$</td>
<td>Total electron and hole concentrations (may be functions of time and/or position)</td>
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<td>$\delta n = n - n_0$</td>
<td>Excess electron concentration (may be function of time and/or position)</td>
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<td>$\tau_{n0}, \tau_{p0}$</td>
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Table 6.1: Notations used in Carrier Generation and Recombination
Excess Carrier Generation and Recombination

- A steady-state generation of excess electrons and holes will not cause a continual buildup of the carrier concentrations. As in the case of thermal equilibrium, an electron in the conduction band may "fall down" into the valence band, leading to the process of excess electron-hole recombination.

- The recombination rate for excess electrons is denoted by $R_n'$, and for excess holes by $R_p'$. Both parameters have units of $/#\text{cm}^3\text{-s}$. The excess electrons and holes recombine in pairs, so the recombination rates must be equal. We can then write $R_n' = R_p'$.

- In the direct band-to-band recombination that we are considering, the recombination occurs spontaneously: thus, the probability of an electron and hole recombining is constant with time.

- The rate at which electrons recombine must be proportional to the electron concentration and must also be proportional to the hole concentration.

- If there are no electrons or holes, there can be no recombination.
Excess Carrier Generation and Recombination

- The net rate of change in the electron concentration can be written as:

\[
\frac{dn(t)}{dt} = \alpha_r [n_i^2 - n(t)p(t)]
\]

(eq. 6.1)

where \( \alpha_r n_i^2 \) (eq. 6.1) is thermal-equilibrium generation rate.

- Since excess electrons and holes are created and recombine in pairs, \( \delta n(t) = \delta p(t) \). (Excess electron and hole concentrations are equal and termed excess carriers.)

- The thermal-equilibrium parameters, \( n_0 \) and \( p_0 \), independent of time, become: (eq. 6.2)

\[
\frac{d(\delta n(t))}{dt} = \alpha_r [n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t))] = -\alpha_r \delta n(t)[(n_0 + p_0) + \delta n(t)]
\]
Excess Carrier Generation and Recombination

- (Eq. 6.2) can easily be solved if we impose the condition of **Low Level Injection**.

- In an extrinsic n-type material, we generally have $n_0 >> p_0$ and, in an extrinsic p-type material, we generally have $p_0 >> n_0$.

- Low-level injection means that the excess carrier concentration is much less than the thermal equilibrium majority carrier concentration.

- Conversely, high-level injection occurs when the excess carrier concentration becomes comparable to or greater than the thermal equilibrium majority carrier concentrations.

- If we consider a p-type material ($p_0 >> n_0$) under low-level injection ($\delta n(t) << p_0$), then (eq. (6.2)) is:

$$\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t)$$

(eq. (6.2))
Excess Carrier Generation and Recombination

- The solution to the equation is an exponential decay from the initial excess concentration, or

\[ \delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_{n0}} \]  

(eq. 6.3)

where \( \tau_{n0} = (\alpha_r p_0)^{-1} \) and is a constant for the low-level injection.

- The above equation describes the decay of excess minority carrier electrons so that \( \tau_{n0} \) is often referred to as the excess minority carrier lifetime.
Excess Carrier Generation and Recombination

- The recombination rate defined as a positive quantity--of excess minority carrier electrons can be written as:

\[ \dot{R}_n = -\frac{d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta(n(t))}{\tau_{n0}} \]

- For the direct band-to-band recombination, the excess majority carrier holes recombine at the same rate, so that for the p-type material

\[ R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}} \]

- For an n-type material \((n_0 >> p_0)\) under low-level injection \((\delta n(t) << n_0)\), the decay of minority carrier holes occurs with a time constant \(\tau_{p0} = (\alpha_r n_0)^{-1}\) where \(\tau_{p0}\) is also referred to as the excess minority carrier lifetime. The recombination rate of the majority carrier electrons is the same as that of the minority carrier holes, so we have

\[ R'_n = R'_p = \frac{\delta p(t)}{\tau_{p0}} \]

- The generation rates of excess carriers are not functions of electron or hole concentrations. In general, the generation and recombination rates may be functions of the space coordinates and time.
Exercise

1. Consider a semiconductor in which $n_0 = 10^{13}$ cm$^{-3}$ and $n_i = 10^{10}$ cm$^{-3}$. Assume that the excess-carrier lifetime is $10^{-6}$ s. Determine the electron-hole recombination rate if the excess-hole concentration is $\delta p = 5 \times 10^{13}$ cm$^{-3}$.

$$R_n' = R_p' = \frac{\delta p(t)}{\tau_{p0}}$$
Solution

1. n-type semiconductor, low-injection so that

\[
R' = \frac{\delta p}{\tau_p} = \frac{5 \times 10^{13}}{10^{-6}} = 5 \times 10^{19} \text{ cm}^{-3} \text{s}^{-1}
\]
Exercise

2. A semiconductor in thermal equilibrium, has a hole concentration of $p_0 = 10^{16}$ cm$^{-3}$ and an intrinsic concentration of $n_i = 10^{10}$ cm$^{-3}$. The minority carrier lifetime is $2 \times 10^{-7}$ s.

(a) Determine the thermal-equilibrium recombination rate of electrons.

(b) Determine the change in the recombination rate of electrons if an excess electron concentration of $\delta n = 10^{12}$ cm$^{-3}$ exists.

$$
\dot{R}_n = \dot{R}_p = \frac{\delta n(t)}{\tau_{n0}}
$$
Solution

a) thermal-equilibrium recombination rate of electrons

\[ R_{n0} = \frac{n_0}{\tau_{n0}} \]

and

\[ n_0 = \frac{n_i}{p_0} = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ cm}^{-3} \]

Then

\[ R_{n0} = \frac{(10^4)}{(2 \times 10^{-7})} = 5 \times 10^{10} \text{ cm}^{-3} \text{ s}^{-1} \]
Solution

b) the change in the recombination rate of electrons if an excess electron concentration of $\delta n = 10^{12} \text{ cm}^{-3}$ exists

$$R_{n0} = \frac{\delta n}{\tau_{n0}} = \frac{10^{12}}{2 \times 10^{-7}} = 5 \times 10^{18} \text{ cm}^{-3} \text{ s}^{-1}$$

Therefore,

$$\Delta R_n = R_n - R_{n0} = 5 \times 10^{18} - 5 \times 10^{10}$$

$$\Delta R_n = 5 \times 10^{18} \text{ cm}^{-3} \text{ s}^{-1}$$
Continuity Equations

- The continuity equations for electrons and holes are shown as a differential volume element in which a one-dimensional hole particle flux is entering the differential element at $r$ and is leaving at $x + dx$.

- The parameter $F_{px}^+$ is the hole-particle flux, or flow, and has units of number of holes/cm$^2$-s.

- For the $x$ component of the particle current density:

$$F_{px}^+(x + dx) = F_{px}^+(x) + \frac{\partial F_{px}^+}{\partial x}.dx$$

Fig. 6.2: Differential volume showing $x$ component of the hole-particle flux.
Continuity Equations

- This equation is a Taylor expansion of $F_{px}^+(x + dx)$. where the differential length $dx$ is small, so that only the first two terms in the expansion are significant. The net increase in the number of holes per unit time within the differential volume element due to the $x$-component of hole flux is given by:

$$\frac{\partial p}{\partial t} dxdydz = [F_{px}^+(x) - F_{px}^+(x + dx)] dydz = -\frac{\partial F_{px}^+}{\partial x} dxdydz$$

- If $F_{px}^+(x) > F_{px}^+(x + dx)$, for example, there will be a net increase in the number of holes in the differential volume element with time.
Continuity Equations

- The generation rate and recombination rate of holes will also affect the hole concentration in the differential volume. The net increase in the number of holes per unit time in the differential volume element is then given by

\[
\frac{\partial p}{\partial t} \, dx \, dy \, dz = - \frac{\partial F^+_p}{\partial x} \, dx \, dy \, dz + g \, p \, dx \, dy \, dz - \frac{P}{\tau_{pt}} \, dx \, dy \, dz
\]

where \( p \) is the density of holes.

- The first term on the right side of the equation is the increase in the number of holes per unit time due to the hole flux, the second term is the increase in the number of holes per unit time due to the generation of holes, and the last term is the decrease in the number of holes per unit time due to the recombination of holes.

- The recombination rate for holes is given by \( p/\tau_{pt} \) where \( \tau_{pt} \) includes the thermal equilibrium carrier lifetime and the excess carrier lifetime.
Continuity Equations

- If we divide both sides of the equation by the differential volume $dx\,dy\,dz$, the net increase in the hole concentration per unit time is:

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pl}}$$

- The above equation is known as the continuity equation for holes.

- Similarly, the one-dimensional continuity equation for electrons is given by:

$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^+}{\partial x} + g_n - \frac{n}{\tau_{nl}}$$

where $F_n^-$ is the electron-particle flow, or flux, also given in units of number of electrons/cm$^2$-s.
Time-Dependent Diffusion Equations

- The hole and electron current densities are given as:
  \[
  J_p = e\mu_p pE - eD_p \frac{\partial p}{\partial x}
  \]
  \[
  J_n = e\mu_n nE - eD_n \frac{\partial n}{\partial x}
  \]

- Dividing the hole current density by \((+e)\) and the electron current density by \((-e)\) to find the flux of each particle:
  \[
  \frac{J_p}{(+e)} = F_p^+ = \mu_p pE - D_p \frac{\partial p}{\partial x}
  \]
  \[
  \frac{J_n}{(-e)} = F_n^+ = \mu_n nE - D_n \frac{\partial n}{\partial x}
  \]
Time-Dependent Diffusion Equations

- Taking the divergence of the equations, and substituting back into the continuity equations:

\[ \frac{\partial p}{\partial t} = -\mu_p \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}} \]

\[ \frac{\partial n}{\partial t} = -\mu_n \frac{\partial (nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}} \]

- Keeping in mind that we are limiting ourselves to a one-dimensional analysis, we can expand the derivative of the product as

\[ \frac{\partial (pE)}{\partial x} = E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \]
Time-Dependent Diffusion Equations

- In a more generalized three-dimensional analysis, the above equation would have to be replaced by a vector identity.

\[ D_p \frac{\partial^2 p}{\partial x^2} - \mu_p (E \frac{\partial (p)}{\partial x} + p \frac{\partial (E)}{\partial x}) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial p}{\partial t} \]

\[ D_n \frac{\partial^2 n}{\partial x^2} - \mu_n (E \frac{\partial (n)}{\partial x} + n \frac{\partial (E)}{\partial x}) + g_n - \frac{n}{\tau_{pt}} = \frac{\partial n}{\partial t} \]

- The above equations are the time-dependent diffusion equations for holes and electrons, respectively. Since both the hole concentration \( p \) and the electron concentration \( n \) contain the excess concentrations, the equations describe the space and time behavior of the excess carriers.
Time-Dependent Diffusion Equations

- The hole and electron concentrations are functions of both the thermal equilibrium. The thermal equilibrium concentrations, \( n_o \) and \( p_o \), are not functions of time.

- For the special case of a homogeneous semiconductor, \( n_o \) and \( p_o \) are also independent of the space coordinates, therefore:

\[
D_p \frac{\partial^2 (\delta p)}{\partial x^2} - \mu_p (E \frac{\partial (\delta p)}{\partial x} + p \frac{\partial (E)}{\partial x}) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial (\delta p)}{\partial t}
\]

\[
D_n \frac{\partial^2 (\delta n)}{\partial x^2} - \mu_n (E \frac{\partial (\delta n)}{\partial x} + n \frac{\partial (E)}{\partial x}) + g_n - \frac{n}{\tau_{pt}} = \frac{\partial (\delta n)}{\partial t}
\]

- Note that the Equations (6.29) and (6.30) contain terms involving the total concentrations, \( p \) and \( n \), and terms involving only the excess concentrations, \( \delta p \) and \( \delta n \).
Ambipolar Transport

- If a pulse of excess electrons and a pulse of excess holes are created at a particular point in a semiconductor with an applied electric field, the excess holes and electrons will tend to drift in opposite directions.

- Electrons and holes are charged particles, any separation will induce an internal electric field between the two sets of particles. This internal electric field will create a force attracting the electrons and holes back toward each other.

- The electric field is then composed of the externally applied field plus the induced internal field.

\[ E = E_{app} + E_{int} \]

Fig. 6.3: The creation of an internal electric field as excess electrons and holes tend to separate.
Ambipolar Transport

- Since the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and excess holes together.
- The negatively charged electrons and positively charged holes then will drift or diffuse together with a single effective mobility or diffusion coefficient.
- This phenomenon is called **ambipolar diffusion** or **ambipolar transport**:

\[
D \frac{\partial^2 (\delta n)}{\partial x^2} + \mu E \frac{\partial (\delta n)}{\partial x} + g - R = \frac{\partial (\delta n)}{\partial t}
\]
The ambipolar diffusion coefficient $D'$ and the ambipolar mobility coefficient $\mu'$ are defined from Einstein’s relation as follows:

$$D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

**Applications of Ambipolar Transport**: Illustration of the behavior of excess carriers in semiconductors, in PN junction diodes and other devices.
Exercise

3. Consider an infinitely large, homogeneous n-type semiconductor with a zero applied electric field. Assume that, for t < 0, the semiconductor is in thermal equilibrium and that, for t≥0, a uniform generation rate exists in the crystal.

Calculate the excess carrier concentration as a function of time assuming the condition of low injection (where excess carrier concentration is much lower than thermal equilibrium majority carrier concentration δn<<p₀ (p type) or δp<<n₀ (n type) ).

\[ D \frac{\partial^2 (\delta n)}{\partial x^2} + \mu E \frac{\partial (\delta n)}{\partial x} + g - R = \frac{\partial (\delta n)}{\partial t} \]
The condition of a uniform generation rate and a homogeneous semiconductor implies that:

\[ \frac{\partial^2 (\delta p)}{\partial x^2} = \frac{\partial (\delta p)}{\partial x} = 0 \]

Therefore, the ambipolar transport equation reduces to:

\[ g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial (\delta p)}{\partial t} \]

The solution of the differential is:

\[ \delta p(t) = g' \tau_{p0} (1 - e^{-\frac{t}{\tau_{p0}}}) \]
Exercise

4. Consider an n-type Silicon at $T = 300\text{K}$ doped at $N_d = 2 \times 10^{16}\text{cm}^{-3}$. Assume that $\tau_{p0} = 10^{-7}\text{s}$ and $g' = 5 \times 10^{21}\text{cm}^{-3}\text{s}^{-1}$. Determine the time dependence of excess carriers in reaching a steady-state condition. Does low injection apply?

\[
\delta p(t) = g' \tau_{p0} (1 - e^{-\frac{t}{\tau_{p0}}})
\]
Solution

4. Using the equation:

\[ \delta p(t) = g' \tau_{p0} (1 - e^{-\frac{t}{\tau_{p0}}}) \]

Therefore,

\[ \delta p(t) = (5 \times 10^{21}) - (10^{-7})(1 - e^{-\frac{t}{10^{-7}}}) = 5 \times 10^{14} [1 - e^{\frac{-t}{10^{-7}}}] \text{cm}^{-3} \]

Note 1: for \( t \equiv \infty \), the steady state excess hole and electron concentration of \( 5 \times 10^{14} \text{cm}^{-3} \) exists.

Note 2: \( \delta p \ll n_0 \) therefore low injection is valid.
Dielectric Relaxation Time Constant

- **Quasi-neutrality condition**: the concentration of excess holes is balanced by an equal concentration of excess electrons.

- If a uniform concentration of holes $\delta p$ is suddenly injected into a portion of the surface of a semiconductor, we will instantly have a concentration of excess holes and a net positive charge density that is not balanced by a concentration of excess electrons.

**How is charge neutrality achieved and how fast?**

- **Dielectric Relaxation Time Constant**

Fig. 6.4: The injection of a concentration of holes into a small region at the surface of an n-type semiconductor
There are three defining equations to be considered. Poisson's equation is: (gradient of electric field equals net charge density divided by permittivity) $\nabla . E = \frac{\rho}{\varepsilon}$

From Ohm’s Law: (current density equals conductivity times electric field) $J = \sigma E$

And from continuity equation, neglecting the effects of generation and recombination: $\nabla . J = - \frac{\partial \rho}{\partial t}$

The parameter $\rho$ is the net charge density and the initial value is given by $e(\delta p)$. We will assume that $\delta p$ is uniform over a short distance at the surface. The parameter $\varepsilon$ is the permittivity of the semiconductor.
Dielectric Relaxation Time Constant

- Taking the divergence of Ohm's law and using Poisson's equation:
  \[ \nabla \cdot J = \sigma \nabla \cdot E = \frac{\sigma \rho}{\varepsilon} \]

- Substituting the above equation into the continuity equation, which can be rearranged into the first order equation:
  \[ \frac{d \rho}{dt} = -\frac{\sigma}{\varepsilon} \frac{\partial \rho}{\partial t} = -\frac{d \rho}{dt} \]

  Whose solution is:
  \[ \rho(t) = \rho(0) e^{-\frac{t}{\tau_d}} \]

Dielectric Relaxation Time Constant is:

\[ \tau_d = \frac{\varepsilon}{\sigma} \]
Exercise

5. Assume an n-type Silicon semiconductor with a donor impurity concentration of $N_d = 10^{16} \text{cm}^{-3}$ and mobility $\mu_n = 1200 \text{cm}^2/\text{V} \cdot \text{sec}$.

- Calculate the dielectric relaxation time constant.
Solution

5. The conductivity is found as

\[ \sigma \approx e \mu_n N_d = (1.6 \times 10^{-19}) (1200) (10^{16}) = 1.92 (\Omega cm)^{-1} \]

- The permittivity of Silicon is calculated as:

\[ \varepsilon = \varepsilon_r \varepsilon_0 = (11.7)(8.85 \times 10^{-14}) F / cm \]

The dielectric relaxation time constant is then:

\[ \tau_d = \frac{(11.7)(8.85 \times 10^{-14}) F / cm}{(1.92 \Omega cm^{-1})} = 5.39 \times 10^{-13} s \]
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